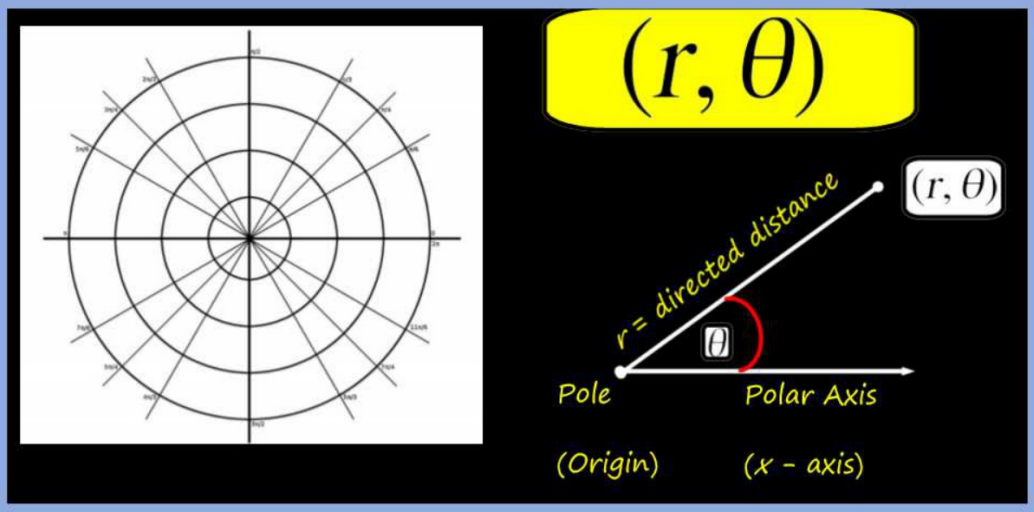
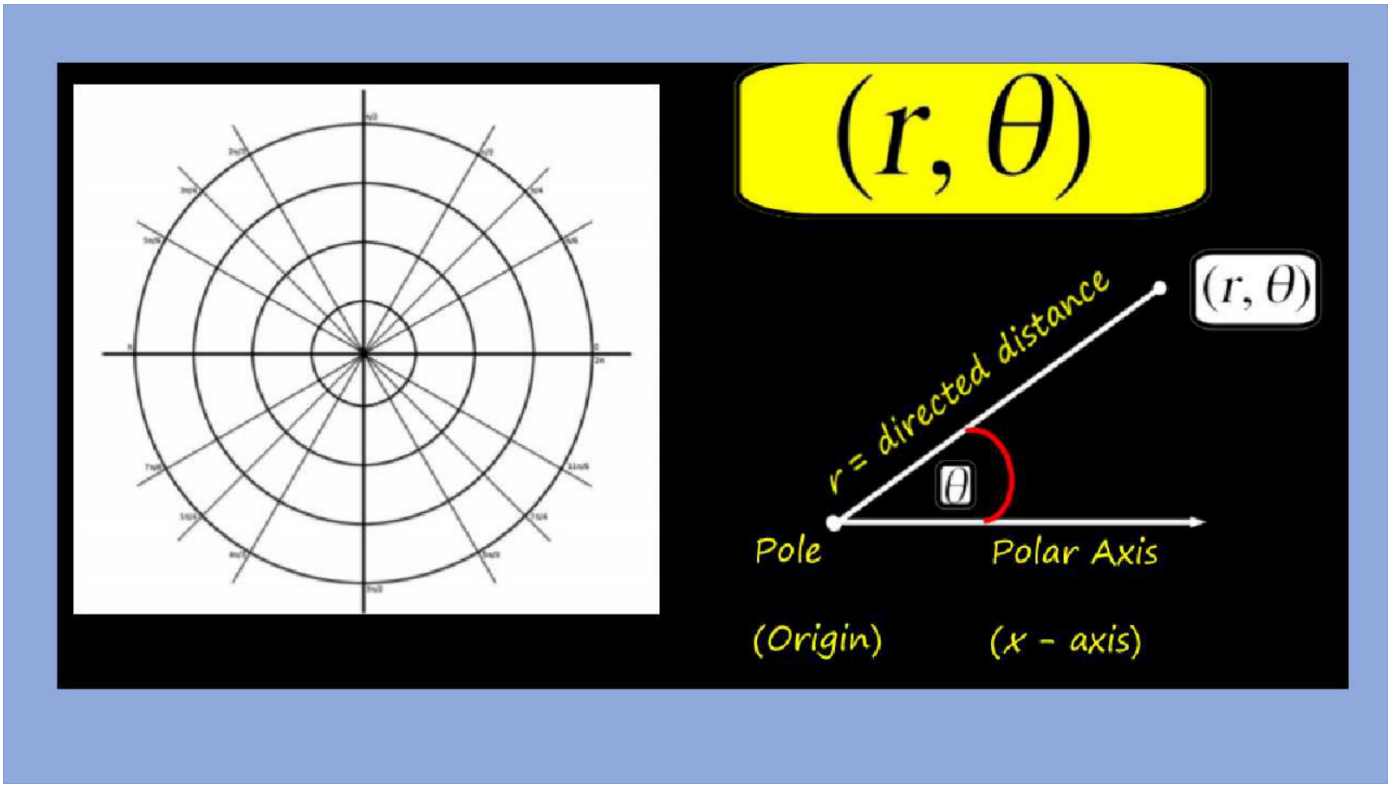
10.7 Polar Coordinates

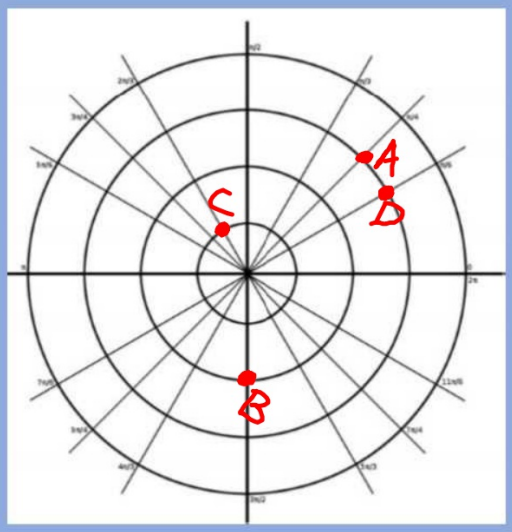


Definitions/ Characteristics

- Polar coordinates: a 2nd way to plot points using a directed angle and a directed distance.
- Written as: (r, Θ) where r is the radius (distance) and Θ is the angle
- Uses concentric circles and angles in standard position
- Each point can have ~~inf~~ many different polar coordinates to represent it (unlike rectangular coordinates- which only have 1)


(r, θ)
(r, θ)
r = directed distance
θ
Pole
Polar Axis
(Origin)
(x - axis)

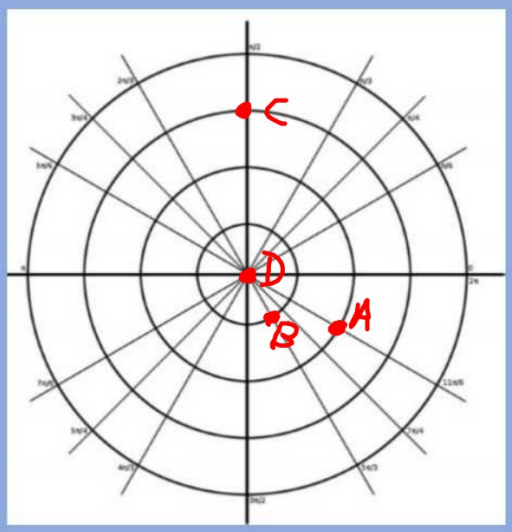
Graphing Polar Coordinates



$A\left(3, \frac{\pi}{4}\right)$ $B\left(-2, \frac{\pi}{2}\right)$

$C\left(1, -\frac{4\pi}{3}\right)$ $D\left(-3, -\frac{5\pi}{6}\right)$

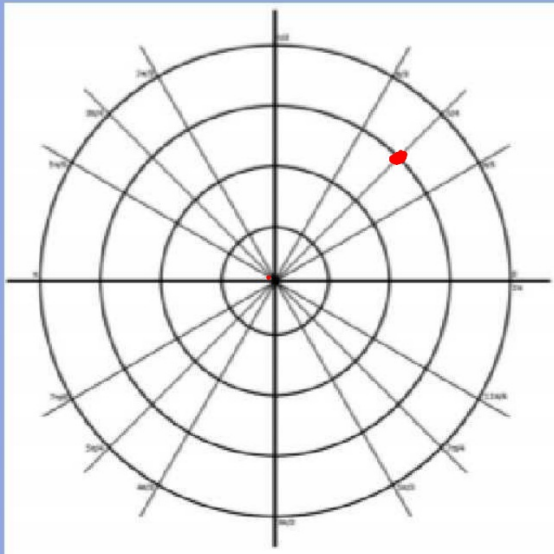
Graphing Polar Coordinates



$A\left(2, -\frac{\pi}{6}\right)$ $B\left(-1, -\frac{4\pi}{3}\right)$

$C\left(-3, \frac{3\pi}{2}\right)$ $D\left(0, \frac{13\pi}{16}\right)$

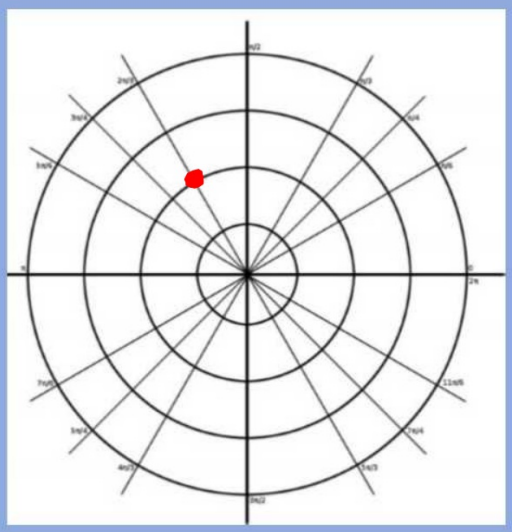
Multiple Representations of Polar Coordinates



$\left(3, \frac{\pi}{4}\right)$ $\left(3, -\frac{7\pi}{4}\right)$

$\left(-3, -\frac{3\pi}{4}\right)$ $\left(-3, \frac{5\pi}{4}\right)$

Multiple Representations of Polar Coordinates



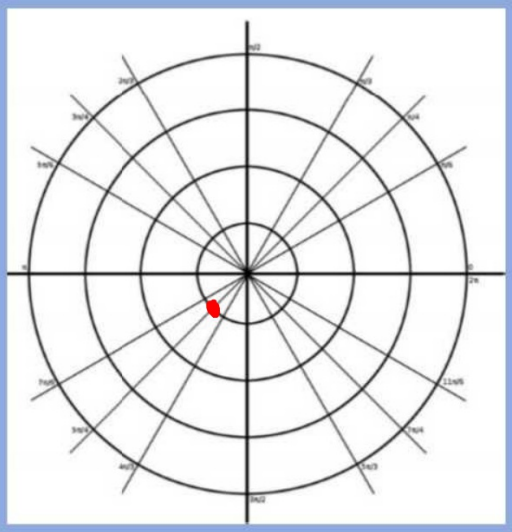
Graph the point

$$\left(2, \frac{2\pi}{3}\right)$$

Determine 3 different coordinates that map to the same location:

- $\left(2, -\frac{4\pi}{3}\right)$
- $\left(-2, -\pi/3\right)$
- $\left(-2, \frac{5\pi}{3}\right)$

Multiple Representations of Polar Coordinates



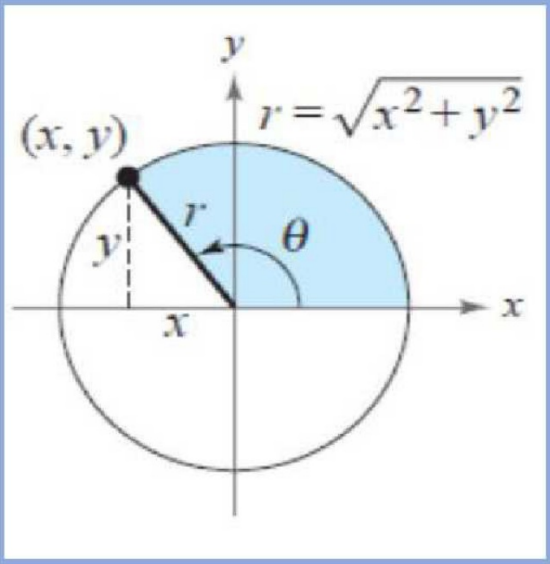
Graph the point

$$\left(-1, \frac{\pi}{4}\right)$$

Determine 3 different coordinates that map to the same location:

- $\left(-1, -7\pi/4\right)$
- $\left(1, 5\pi/4\right)$
- $\left(1, -3\pi/4\right)$

Conversions



Converting: Polar to Rectangular Coordinates

$$x = r\cos(\theta) \qquad y = r\sin(\theta)$$

Determine the Rectangular Coordinates of the given points.

$$\left(\overset{r}{2}, \overset{\theta}{\frac{\pi}{3}}\right) \quad \boxed{(1, \sqrt{3})}$$

$x = r \cdot \cos\theta$

$x = 2 \cdot \cos(\pi/3)$

$x = 2\left(\frac{1}{2}\right)$

$x = 1$

$y = r \cdot \sin\theta$

$x = 2 \cdot \sin(\pi/3)$

$y = 2 \cdot \left(\frac{\sqrt{3}}{2}\right)$

$y = \sqrt{3}$

$$\left(-5, \frac{11\pi}{6}\right) \quad \boxed{\left(\frac{-5\sqrt{3}}{2}, \frac{5}{2}\right)}$$

$x = r \cdot \cos\theta$

$x = -5 \cdot \cos\left(\frac{11\pi}{6}\right)$

$x = -5\left(\frac{\sqrt{3}}{2}\right)$

$x = \frac{-5\sqrt{3}}{2}$

$y = r \cdot \sin\theta$

$y = -5 \cdot \sin\left(\frac{11\pi}{6}\right)$

$y = -5\left(-\frac{1}{2}\right)$

$y = 5/2$

Converting: Polar to Rectangular Coordinates

$$x = r\cos(\theta) \qquad y = r\sin(\theta)$$

Determine the Rectangular Coordinates of the given points.

$(4, 3\pi)$ $\boxed{(-4, 0)}$

$x = r \cdot \cos\theta$
$x = 4 \cdot \cos(3\pi)$
$x = 4(-1)$
$x = -4$

$y = r \cdot \sin\theta$
$y = 4 \cdot \sin(3\pi)$
$y = 4(0)$
$y = 0$

$\left(-2, -\frac{5\pi}{6}\right)$ $\frac{7\pi}{6}$ $\boxed{(\sqrt{3}, 1)}$

$x = r \cdot \cos\theta$
$x = -2 \cdot \cos\left(-\frac{5\pi}{6}\right)$
$x = -2 \cdot -\frac{\sqrt{3}}{2}$
$x = \sqrt{3}$

$y = r \cdot \sin\theta$
$y = -2 \cdot \sin\left(-\frac{5\pi}{6}\right)$
$y = -2\left(-\frac{1}{2}\right)$
$y = 1$

Converting: Rectangular to Polar Coordinates

$$\tan(\theta) = \frac{y}{x}$$

$$x^2 + y^2 = r^2$$

Determine the Polar Coordinates of the given points.

1st quad

$(3, 3\sqrt{3})$ $\boxed{(6, \pi/3)}$

$x^2 + y^2 = r^2$

$3^2 + (3\sqrt{3})^2 = r^2$

$9 + 27 = r^2$

$36 = r^2$

$6 = r$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{3\sqrt{3}}{3}$

$\tan\theta = \sqrt{3}$

$\theta = \pi/3$

2nd quad

$(-4, 4)$ $\boxed{(4\sqrt{2}, 3\pi/4)}$

$x^2 + y^2 = r^2$

$16 + 16 = r^2$

$32 = r^2$

$4\sqrt{2} = r$

$\tan\theta = \frac{y}{x}$

$\tan\theta = 4/-4$

$\tan\theta = -1$

$\theta = \pi/4$

$\theta = 3\pi/4$

Converting: Rectangular to Polar Coordinates

$$\tan(\theta) = \frac{y}{x}$$

$$x^2 + y^2 = r^2$$

Determine the Polar Coordinates of the given points.

2nd

$(-1, 1)$ $\boxed{(\sqrt{2}, 3\pi/4)}$

$x^2+y^2=r^2$

$1+1=r^2$

$2=r^2$

$r=\sqrt{2}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{1}{-1} = -1$

$\theta = \pi/4$

$\downarrow$

$3\pi/4$

4th quad

$(2\sqrt{3}, -2)$ $\boxed{(4, 11\pi/6)}$

$x^2+y^2=r^2$

$(2\sqrt{3})^2+(-2)^2=r^2$

$12+4=r^2$

$16=r^2$

$4=r$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-2}{2\sqrt{3}}$

$\tan\theta = -\sqrt{3}/3$

$\theta = \frac{\pi}{6} \rightarrow \frac{11\pi}{6}$

Converting Equations: Rectangular to Polar

- Steps:
  - Substitute for x and y
  - Solve for "r"

$2x - 3y = 1$

$2(r\cos\theta) - 3(r\sin\theta) = 1$

$2r\cos\theta - 3r\sin\theta = 1$

$r(2\cos\theta - 3\sin\theta = 1$

$$\boxed{r = \frac{1}{2\cos\theta - 3\sin\theta}}$$

$x^2 + y^2 = 4$

$(r\cos\theta)^2 + (r\sin\theta)^2 = 4$

$r^2\cos^2\theta + r^2\sin^2\theta = 4$

$r^2(\cos^2\theta + \sin^2\theta) = 4$

$r^2(1) = 4$

$r^2 = 4$

$\boxed{r = 2}$

$r^2 = 4$

$\boxed{r = -2}$

Converting Equations: Rectangular to Polar

$2xy = 10$

$2(r\cos\theta)(r\sin\theta) = 10$

$2r^2\cos\theta\sin\theta = 10$

$r^2\cos\theta\sin\theta = 5$

$r^2 = \dfrac{5}{\cos\theta\sin\theta}$

$r^2 = 5\sec\theta\csc\theta$

$x^2 + y^2 - 8x = 0$

$r^2 - 8r\cos\theta = 0$

$r^2 = 8r\cos\theta$

Divide by "r"

$\boxed{r = 8\cos\theta}$

Converting Equations: Polar to Rectangular

- Must think to solve these- manipulate the equation to match with one of the four equations used to convert points

1.) r = 2

$r^2 = 2^2$

$\boxed{x^2 + y^2 = 4}$

2.) $\theta = \frac{\pi}{3}$

$\tan\theta = \tan(\pi/3)$

$\frac{y}{x} = \sqrt{3}$

$\boxed{y = \sqrt{3} \cdot x}$

3.) $r = sec\theta$

$r = \frac{1}{\cos\theta}$

$r\cos\theta = 1$

$\boxed{x = 1}$

More examples

4.) r = 1

$r^2 = 1^2$

$\boxed{x^2 + y^2 = 1}$

5.) $\theta = \frac{\pi}{4}$

$\tan\theta = \tan \pi/4$

$\frac{y}{x} = 1$

$\boxed{y = x}$

6.) $r = csc\theta$

$r = \frac{1}{\sin\theta}$

$r\sin\theta = 1$

$\boxed{y = 1}$

Practice problems

- Pg 783 # 1-4, 6, 10-14 even, 18-24 even, 34-42 even, 50-56 even

HMWK

- Pg 783 # 9-13 odd, 17-23 odd, 33-41 odd, 49-55 odd